3-1 Solving Systems Using Tables and Graphs

Common Core State Standards

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
Also A-REI.C.6, A-CED.D.11, A-CED.A.3

MP 1, MP 2, MP 3, MP 4, MP 5

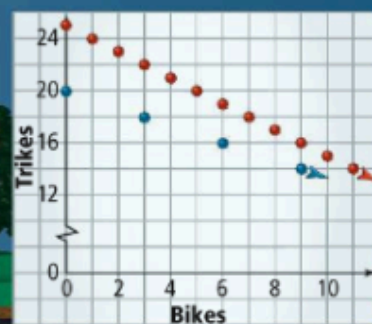
Objective To solve a linear system using a graph or a table

If there were only 2 bikes and 2 trikes how many wheels would there be?

MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

There are 25 bikes and trikes at the park. The bikes and trikes have 60 wheels in all. In the graph, the red dots show sums of 25. The blue dots show 60-wheel combinations. How many bikes and trikes are in the park? Explain.



Lesson Vocabulary

- system of equations
- linear system
- solution of a system
- inconsistent system
- consistent system
- independent system
- dependent system

When you have two or more related unknowns, you may be able to represent their relationship with a **system of equations**—a set of two or more equations.

Essential Understanding To solve a system of equations, find a set of values that replace the variables in the equations and make each equation true.

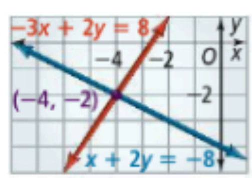
A **linear system** consists of linear equations. A **solution of a system** is a set of values for the variables that makes all the equations true. You can solve a system of equations graphically or by using tables.

Problem 1 Using a Graph or Table to Solve a System

Think

How can you use a graph to find the solution of a system?
Find the point where the two lines intersect.

What is the solution of the system? $\begin{cases} -3x + 2y = 8 \\ x + 2y = -8 \end{cases}$



Method 1 Graph the equations. The point of intersection appears to be $(-4, -2)$.

Check by substituting the values into both equations.

$-3x + 2y = 8 \qquad x + 2y = -8$

$-3(-4) + 2(-2) = 8$ ✔ $\qquad -4 + 2(-2) = -8$ ✔

Both equations are true so $(-4, -2)$ is the solution of the system.

Method 2 Use a table. Write the equations in slope-intercept form.

$$-3x + 2y = 8 \qquad\qquad x + 2y = -8$$
$$2y = 3x + 8 \qquad\qquad 2y = -x - 8$$
$$y_1 = \frac{3}{2}x + 4 \qquad\qquad y_2 = -\frac{1}{2}x - 4$$

X	Y_1	Y_2
−5	−3.5	−1.5
−4	−2	−2
−3	−.5	−2.5
−2	1	−3
−1	2.5	−3.5
0	4	−4
1	5.5	−4.5
X = −4		

Enter the equations in the **Y=** screen as **Y1** and **Y2**.
View the table. Adjust the x-values until you see $y_1 = y_2$.

When $x = -4$, both y_1 and y_2 equal −2. So, (−4, −2) is the solution of the system.

Got It? **1.** What is the solution of the system? $\begin{cases} x - 2y = 4 \\ 3x + y = 5 \end{cases}$

Problem 2 Using a Table to Solve a Problem STEM

Biology **The diagrams show the birth lengths and growth rates of two species of shark. If the growth rates stay the same, at what age would a Spiny Dogfish and a Greenland shark be the same length?**

Step 1 Define the variables and write the equation for the length of each shark.

Let x = age in years.
Let y = length in centimeters.

Length of Greenland: $y_1 = 0.75x + 37$
Length of Spiny Dogfish: $y_2 = 1.5x + 22$

Step 2 Use the table to solve the problem.

List x-values until the corresponding y-values match.

Shark Length in cm

Age	Greenland	Spiny Dogfish
x	$y_1 = 0.75x + 37$	$y_2 = 1.5x + 22$
15	48.25	44.5
16	49	46
20	52	52

The sharks will be the same length when they are 20 years old.

Think
How can you use slope-intercept form to write each equation?
Use the growth rate for m and the length at birth for b.

Got It? **2. a.** If the growth rates continue, how long will each shark be when it is 25 years old?

b. Reasoning Explain why growth rates for these sharks may not continue indefinitely.

Problem 3 Using Linear Regression

Population **The table shows the populations of the New York City and Los Angeles metropolitan regions from the census reports for 1950 through 2000. Assuming these linear trends continue, when will the populations of these regions be equal? What will that population be?**

Populations of New York City and Los Angeles Metropolitan Regions (1950–2000)

	1950	1960	1970	1980	1990	2000
New York City	12,911,994	14,759,429	16,178,700	16,121,297	18,087,251	21,199,865
Los Angeles	4,367,911	6,742,696	7,032,075	11,497,568	14,531,529	16,373,645

SOURCE : U.S. Census Bureau

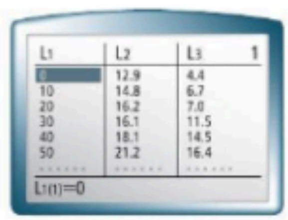
Know
Population data for two regions

Need
The point in time when their populations will be the same

Plan
- Use a calculator to find linear regression models.
- Plot the models.
- Find the point of intersection.

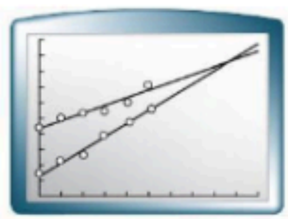
Enter all the numbers as millions, rounded to the nearest hundred thousand. For example, enter 12,911,994 as 12.9.

Step 1 Enter the data into lists on your calculator.
L1: number of years since 1950
L2: New York City populations
L3: Los Angeles populations



Step 2 Use **LinReg(ax + b)** to find lines of best fit.
Use **L1** and **L2** for New York City.
Use **L1** and **L3** for Los Angeles.

Step 3 Graph the linear regression lines.
Use the **Intersect** feature.



The x-value of the point of intersection is about 87, which represents the year 2037. The data suggest that the populations of the New York City and Los Angeles metropolitan regions will each be about 25.6 million in 2037.

Think
What does x represent?
The x-value is the number of years *since* the zero year.

Got It? 3. The table shows the populations of the San Diego and Detroit metropolitan regions. When were the populations of these regions equal? What was that population?

Populations of San Diego and Detroit Metropolitan Regions (1950–2000)

	1950	1960	1970	1980	1990	2000
San Diego	334,387	573,224	696,769	875,538	1,110,549	1,223,400
Detroit	1,849,568	1,670,144	1,511,482	1,203,339	1,027,974	951,270

SOURCE: U.S. Census Bureau

You can classify a system of two linear equations by the number of solutions.

A **consistent system** has at least one solution.

An **inconsistent system** has no solution.

Consistent system → Independent; Dependent

Inconsistent system

An **independent system** has one solution.

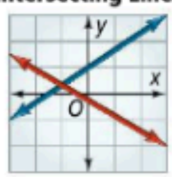
A **dependent system** has infinitely many solutions.

The graphs for an inconsistent system are parallel lines. So, there are no solutions. For a dependent system, the two equations represent the same line.

take note

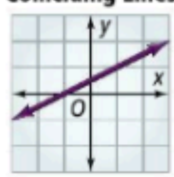
Concept Summary Graphical Solutions of Linear Systems

Intersecting Lines



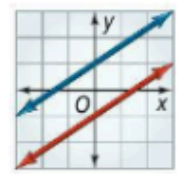
one solution
Consistent
Independent

Coinciding Lines



infinitely many solutions
Consistent
Dependent

Parallel Lines



no solution
Inconsistent

Problem 4 Classifying a System Without Graphing

Plan

What should you compare to classify the system?
Compare the slopes and y-intercepts of each line.

Without graphing, is the system *independent, dependent,* or *inconsistent*?

$$\begin{cases} 4y - 2x = 6 \\ 8y = 4x - 12 \end{cases}$$

Rewrite each equation in slope-intercept form. Compare slopes and y-intercepts.

$4y - 2x = 6$ $\quad\quad$ $8y = 4x - 12$

$y = \frac{1}{2}x + \frac{3}{2}$ $\quad\quad$ $y = \frac{1}{2}x - \frac{3}{2}$

$m = \frac{1}{2}$; y-intercept is $\frac{3}{2}$ $\quad\quad$ $m = \frac{1}{2}$; y-intercept is $-\frac{3}{2}$

The slopes are equal and the y-intercepts are different. the lines are different but parallel. The system is inconsistent.

Got It? 4. Without graphing, is each system *independent, dependent,* or *inconsistent*?

a. $\begin{cases} -3x + y = 4 \\ x - \frac{1}{3}y = 1 \end{cases}$ **b.** $\begin{cases} 2x + 3y = 1 \\ 4x + y = -3 \end{cases}$ **c.** $\begin{cases} y = 2x - 3 \\ 6x - 3y = 9 \end{cases}$

Lesson Check

Do you know HOW?

Solve each system of equations by graphing. Check your solution.

1. $\begin{cases} y = x - 1 \\ y = -x + 3 \end{cases}$

2. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$

3. You bought a total of 6 pens and pencils for \$4. If each pen costs \$1 and each pencil costs \$.50, how many pens and pencils did you buy?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary Is it possible for a system of equations to be both independent and inconsistent? Explain.

5. Open-Ended Write a system of linear equations that has no solution.

6. Reasoning In a system of linear equations, the slope of one line is the negative reciprocal of the slope of the other line. Is this system *independent, dependent,* or *inconsistent*? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each system by graphing or using a table. Check your answers. See Problem 1.

7. $\begin{cases} y = x - 2 \\ y = -2x + 7 \end{cases}$

8. $\begin{cases} y = -x + 3 \\ y = \frac{3}{2}x - 2 \end{cases}$

9. $\begin{cases} 2x + 4y = 12 \\ x + y = 2 \end{cases}$

10. $\begin{cases} x = -3 \\ y = 5 \end{cases}$

11. $\begin{cases} 2x - 2y = 4 \\ y - x = 6 \end{cases}$

12. $\begin{cases} 3x + y = 5 \\ x - y = 7 \end{cases}$

Write and solve a system of equations for each situation. Check your answers. See Problem 2.

13. A store sells small notebooks for \$8 and large notebooks for \$10. If you buy 6 notebooks and spend \$56, how many of each size notebook did you buy?

14. A shop has one-pound bags of peanuts for \$2 and three-pound bags of peanuts for \$5.50. If you buy 5 bags and spend \$17, how many of each size bag did you buy?

Graphing Calculator Find linear models for each set of data. In what year will the two quantities be equal? See Problem 3.

15. **U.S. Life Expectancy at Birth (1970–2000)**

Year	1970	1975	1980	1985	1990	1995	2000
Men (years)	67.1	68.8	70.0	71.1	71.8	72.5	74.3
Women (years)	74.7	76.6	77.4	78.2	78.8	78.9	79.7

SOURCE: U.S. Census Bureau

16. **Annual U.S. Consumption of Vegetables**

Year	1980	1985	1990	1995	1998	1999	2000
Broccoli (lb/person)	1.5	2.6	3.4	4.3	5.1	6.5	6.1
Cucumbers (lb/person)	3.9	4.4	4.7	5.6	6.5	6.8	6.4

SOURCE: U.S. Census Bureau

Without graphing, classify each system as ***independent, dependent,*** or ***inconsistent.***

See Problem 4.

17. $\begin{cases} 7x - y = 6 \\ -7x + y = -6 \end{cases}$

18. $\begin{cases} -3x + y = 4 \\ x - \frac{1}{3}y = 1 \end{cases}$

19. $\begin{cases} 4x + 8y = 12 \\ x + 2y = -3 \end{cases}$

20. $\begin{cases} y = 2x - 1 \\ y = -2x + 5 \end{cases}$

21. $\begin{cases} x = 6 \\ y = -2 \end{cases}$

22. $\begin{cases} 2y = 5x + 6 \\ -10x + 4y = 8 \end{cases}$

23. $\begin{cases} x - 3y = 2 \\ 4x - 12y = 8 \end{cases}$

24. $\begin{cases} y - x = 0 \\ y = -x \end{cases}$

25. $\begin{cases} 2y - x = 4 \\ \frac{1}{2}x + y = 2 \end{cases}$

26. $\begin{cases} x + 4y = 12 \\ 2x - 8y = 4 \end{cases}$

27. $\begin{cases} 4x + 8y = -6 \\ 6x + 12y = -9 \end{cases}$

28. $\begin{cases} 4y - 2x = 6 \\ 8y = 4x - 12 \end{cases}$

B Apply

Graph and solve each system.

29. $\begin{cases} 3 = 4y + x \\ 4y = -x + 3 \end{cases}$

30. $\begin{cases} y = \frac{1}{2}x + \frac{1}{2} \\ y = \frac{1}{4}x + \frac{3}{2} \end{cases}$

31. $\begin{cases} 3x + 6y - 12 = 0 \\ x + 2y = 8 \end{cases}$

32. $\begin{cases} 3x = -5y + 4 \\ 250 + 150x = 300y \end{cases}$

33. $\begin{cases} y = -\frac{1}{2}x + 8 \\ y = 2x - 6 \end{cases}$

34. $\begin{cases} x + 3y = 6 \\ 6y + 2x = 12 \end{cases}$

Without graphing, classify each system as ***independent, dependent,*** or ***inconsistent.***

35. $\begin{cases} 3x - 2y = 8 \\ 4y = 6x - 5 \end{cases}$

36. $\begin{cases} 2x + 8y = 6 \\ x = -4y + 3 \end{cases}$

37. $\begin{cases} 3m = -5n + 4 \\ n - \frac{6}{5} = -\frac{3}{5}m \end{cases}$

38. Reasoning Find the solution of the system of equations $f(x) = 3x - 1$ and $g(x) = |x - 3|$. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are solutions of $3x - 1 = |x - 3|$.

39. Think About a Plan You and a friend are both reading a book. You read 2 pages each minute and have already read 55 pages. Your friend reads 3 pages each minute and has already read 35 pages. Graph and solve a system of equations to find when the two of you will have read the same number of pages. Since the number of pages you have read depends on how long you have been reading, let x represent the number of minutes it takes to read y pages.

- How can you describe the relationship between x and y for you?
- How can you describe the relationship between x and y for your friend?
- How can a graph help you solve this problem?

40. Sports You can choose between two tennis courts at two university campuses to learn how to play tennis. One campus charges $25 per hour. the other campus charges $20 per hour plus a one-time registration fee of $10.

a. Write a system of equations to represent the cost c for h hours of court use at each campus.

b. Graphing Calculator Find the number of hours for which the costs are the same.

c. Reasoning If you want to practice for a total of 10 hours, which university campus should you choose? Explain.

41. Error Analysis Your friend used a graphing calculator to solve a system of linear equations, shown below. After using the **TABLE** feature, your friend says that the system has no solution. Explain what your friend did wrong. What is the solution of the system?

X	Y_1	Y_2
−4	14	10
−3	12	8.5
−2	10	7
−1	8	5.5
0	6	4
1	4	2.5
2	2	1

X=2

$$2x + y = 6 \qquad\qquad 3x + 2y = 8$$
$$y = 6 - 2x \qquad\qquad y = \frac{8 - 3x}{2}$$

42. Reasoning Is it possible for an inconsistent linear system to contain two lines with the same *y*-intercept? Explain.

43. Writing Summarize the possible relationships for the *y*-intercepts, slopes, and number of solutions in a system of two linear equations in two variables.

Reasoning Determine whether each statement is *always, sometimes,* or *never* true for the following system.

$$\begin{cases} y = x + 3 \\ y = mx + b \end{cases}$$

44. If $m = 1$, the system has no solution.

45. If $b = 3$, the system has exactly one solution.

46. If $m \neq 1$, the system has no solution.

47. If $m \neq 1$ and $b = 2$, the system has infinitely many solutions.

Challenge

Open-Ended Write a second equation for each system so that the system will have the indicated number of solutions.

48. infinite number of solutions

$$\begin{cases} \frac{x}{4} + \frac{y}{3} = 1 \\ \underline{\quad ? \quad} \end{cases}$$

49. no solutions

$$\begin{cases} 5x + 2y = 10 \\ \underline{\quad ? \quad} \end{cases}$$

50. Write a system of linear equations with the solution set $\{(x, y) \mid y = 5x + 2\}$.

51. Reasoning What relationship exists between the equations in a dependent system?

52. Economics Research shows that in a certain market only 2000 widgets can be sold at $8 each, but if the price is reduced to $3, then 10,000 can be sold.

a. Let *p* represent price and *n* represent the number of widgets. Identify the independent and dependent variables.

b. Write a linear equation that relates price and the quantity demanded. This type of equation is called a *demand* equation.

c. A shop can make 2000 widgets for $5 each and 20,000 widgets for $2 each. Use this information to write a linear equation that relates price and the quantity supplied. This type of equation is called a *supply* equation.

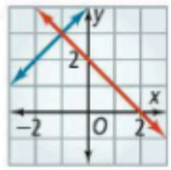
d. Find the equilibrium point where supply is equal to demand. Explain the meaning of the coordinates of this point within the context of the exercise.

Standardized Test Prep

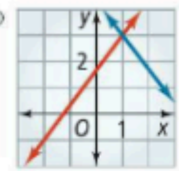
SAT/ACT

53. Which graph shows the solution of the following system? $\begin{cases} 4x + y = 1 \\ x + 4y = -11 \end{cases}$

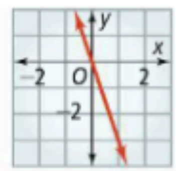
Ⓐ


Ⓑ


Ⓒ


Ⓓ


54. Which is the equation of a line that is perpendicular to the line in the graph?



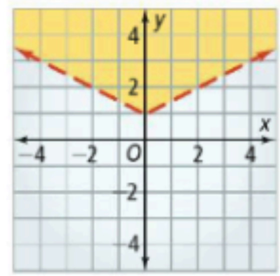
Ⓕ $y = -3x + 2$

Ⓖ $y = \frac{1}{3}x + 5$

Ⓗ $y = -\frac{1}{3}x - 4$

Ⓘ $y = 3x - 1$

55. Which inequality represents the graph at the right?



Ⓐ $y \geq \frac{1}{2}|x| + 1$

Ⓑ $y \leq \frac{1}{2}|x| + 1$

Ⓒ $y > \frac{1}{2}|x| + 1$

Ⓓ $y < \frac{1}{2}|x| + 1$

Extended Response

56. Amy ordered prints of a total of 6 photographs in two different sizes, 5 × 7 and 4 × 6, from an online site. She paid $7.50 for her order. The cost of a 5 × 7 print is $1.75 and the cost of a 4 × 6 print is $.25. Explain how to solve a system of equations using tables to find the number of 4 × 6 prints Amy ordered.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 5

William and Maggie are competing in a triathlon like the one described on page 133. Maggie begins the bicycle portion of the triathlon half an hour ahead of William, and rides at a rate of 12 mi/h. William rides at a rate of 18 mi/h.

a. If the bicycle portion of the race is long enough, can William catch up with Maggie? Explain your answer.

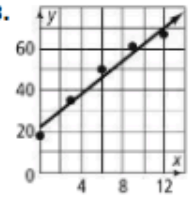
b. Use a table to find how much time it takes William to catch up with Maggie.

c. Write a system of equations to model William's and Maggie's bicycle portions of the triathlon.

d. Graph the system of equations.

e. Does your graph give the same amount of time for William to catch up with Maggie as the table from part (b)? Explain how each tool is used to find the amount of time.

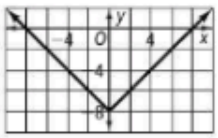
f. After how many miles will William catch up with Maggie?

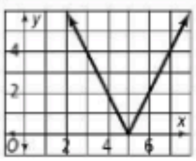
33. 

strong pos. correlation; Answers may vary. Sample: $y = 4x + 22$; 82

34. $y = f(x + 2) - 7$ **35.** $y = -f(x - 5)$
36. $y = f(-x) + 3$ **37.** translated 4 units down
38. vertically stretched by a factor of 12, translated 2 units up **39.** vertically stretched by a factor of 2, reflected across the y-axis, reflected across the x-axis

40. $y = |x - 2| + 4$ **41.** $y = |x + 3|$
42. $y = |x - 5| + 2$ **43.** $y = |x - 4| + 1$

44. 

45. 

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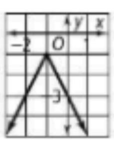
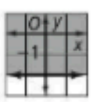
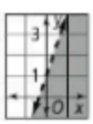
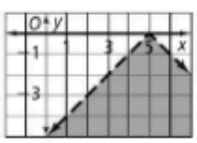
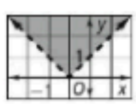
47. 

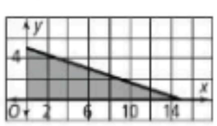
48. (4, 0); $x = 4$
49. (0, 2); $x = 0$

50. 

51. 

52. 

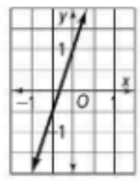
53. 

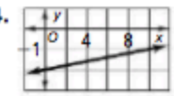
54. a. Answers may vary. Sample: $x + 3y \leq 15$
b. Answers may vary. Sample: domain: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}, range: {0, 1, 2, 3, 4, 5}
c. 

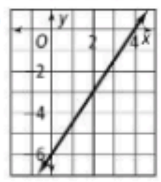
55. Answers may vary. Sample: $y \leq -|x| - 1$

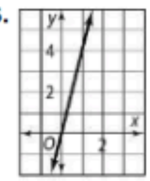
Chapter 3

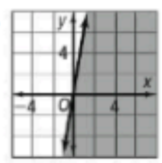
Get Ready! p. 131 **1.** 28 **2.** 33 **3.** $-\frac{15}{2}$ **4.** 15 **5.** $y = \frac{1}{2}x - \frac{5}{2}$ **6.** $y = -2x - 3$ **7.** $y = 5x + 16$ **8.** $y = 3x - 7$ **9.** $y = \frac{2}{5}x - \frac{3}{2}$ **10.** $y = 4x + 11$ **11.** $y = -6x - 8$ **12.** $y = -3x + 18$

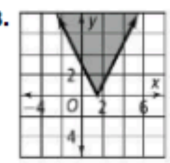
13. 

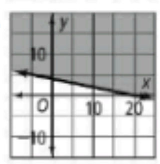
14. 

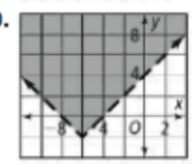
15. 

16. 

17. 

18. 

19. 

20. 

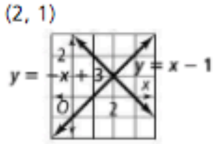
21. Answers may vary. Samples: Rocky Mountains, Appalachian Mountains **22.** Answers may vary. Sample: Your actions are consistent with your words when your actions show what you are saying. For example, if you say you are happy and you are laughing or smiling. **23.** 15 books or more; 15 books or more but less than 19, i.e., 15, 16, 17, or 18 books

Lesson 3-1 pp. 134–141

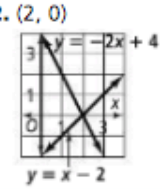
Got It? **1.** (2, −1) **2. a.** Spiny Dogfish: 59.5 cm; Greenland: 55.75 cm **b.** Each species of shark has a maximum total length; growth rates decrease with increase in age. **3.** in the yr 1990; about 1,100,000 **4. a.** inconsistent **b.** independent **c.** dependent

Lesson Check

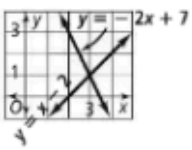
1. (2, 1)



2. (2, 0)

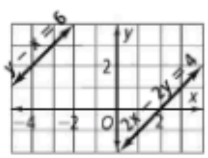


3. 2 pens; 4 pencils **4.** No; an independent system has a unique solution whereas an inconsistent system has no solution.
5. Answers may vary. Sample: $\begin{cases} y = 2x + 1 \\ y = 2x - 3 \end{cases}$
6. Independent; if the slope of one equation is the negative reciprocal of the slope of the other equation, the lines are perpendicular and intersect at a unique point.
Exercises 7–11. How solutions are determined may vary (graphing or using a table).
7. (3, 1)

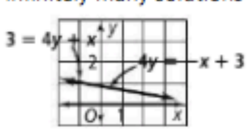


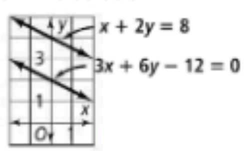
9. (−2, 4)

11. no solution

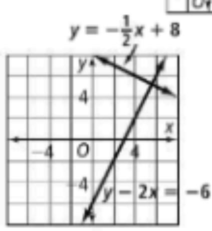


13. 2 small; 4 large
15. Models may vary. Sample: Use 0 for 1970.
$\begin{cases} y = 0.22x + 67.5 \\ y = 0.15x + 75.507 \end{cases}$
Around 2085, the quantities will be equal.
17. dependent **19.** inconsistent **21.** independent
23. dependent **25.** independent **27.** dependent
29. infinitely many solutions **31.** no solution



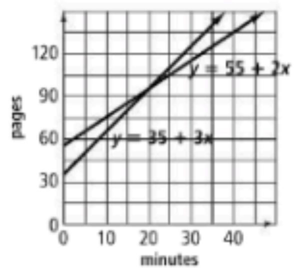


33. $\left(\frac{28}{5}, \frac{26}{5}\right)$



35. inconsistent **37.** inconsistent

39.



After 20 min you and your friend will have read the same number of pages.
41. My friend did not extend the table of values far enough. Scrolling down will show that when $x = 4$, then Y1 = Y2 = −2. So, the system has a solution, (4, −2).
43. An independent system has one solution. The slopes are different, but the *y*-intercepts could be the same. An inconsistent system has no solution. The slopes are the same and the *y*-intercepts are different. A dependent system has an infinite number of solutions. The slopes and *y*-intercepts are the same. **45.** sometimes **47.** never
49. Answers may vary. Sample: $5x + 2y = 5$
51. They are equivalent eqs. **53.** A **55.** C

Lesson 3-2 pp. 142–148

Got It? 1. (−2.5, 2.5) **2.** $.95 per download; $5.50 one-time registration fee **3.** (4, 0) **4. a.** (−2, 3) **b.** Yes; the solution (−5, 2) is a solution to both eqs. in the system, so substituting $y = 2$ into either equation will result in $x = -5$. **5. a.** no solution; The eq. is always false. **b.** infinite number of solutions; The eq. is always true.
Lesson Check 1. (1, 2) **2.** (−6, −6) **3.** (2, 1)
4. (5, −3) **5.** $\left(-\frac{1}{5}, \frac{19}{5}\right)$ **6.** (2, 1)
7. Answers may vary. Sample:
$\begin{cases} 4x - 3y = -2 \\ 3x - 2y = -1 \end{cases}$
$\begin{cases} -8x + 6y = 4 \\ 9x - 6y = -3 \end{cases}$
8. In the substitution method of solving a system of equations, you first solve one equation for one of the variables. Then substitute for this variable in the other equation and solve for the other variable. In the elimination method, you create an equivalent system of equations that contain a pair of additive inverses so that you can eliminate one variable and solve for the remaining variable.
9. Let r = number of regular cups of coffee and c = number of large cups of coffee. First, $r + c = 5$: because a total of 5 cups of coffee were purchased. Second, $r + 1.5c = 6$: because each regular cup of coffee is $1, each large cup is $1.50, and the total spent is $6. Then, solve the system of equations using elimination by